# ON REPEAT SURVEYS IN TWO-STAGE SAMPLING DESIGN

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## 1. Introduction

For a population in which character changes from occasion to occasion, it may be, sometimes, advantageous to repeat the survey on several occasions. Once the decision to study the population on successive occasions is made, several alternatives regarding the sampling fraction to be retained and sampling pattern to be followed do arise. The relative advantages of various types of procedures would depend on the variability among the units and the variability of changes in those units as well as on the relative importance of information on the population means. For an excellent account of the theory of successive sampling in one stage design, the reader is referred to Jessen (1942), Yates (1949), Patternson (1950), Tikkiwal (1951, 55, 56), Eckler (1955), etc. The problems in two stage design have been further explored by Tikkiwal (1965), D. Singh (1968), Singh and Kathuria (1969) and Abraham *et. al.* (1969). In most of the studies made so far, some correlation model for the same units observed on different occasions is assumed, most general one being due to Tikkiwal. Ajgaonkar (1968) has shown that for a unistage sampling design under a specific retention pattern, the results obtained by Patterson, Tikkiwal etc. are true for any correlation pattern. In the present investigation, under a general set up of correlations, an attempt has been made to provide minimum variance unbiased linear estimate of the population mean on any occasion.

The study is confined to following two schemes.

(*a*) Primary sampling units (*p.s.u's.*) are partially retained alongwith their second stage units (*s.s.u's*).

(*b*) All *p.s.u's* are retained and *s.s.u's* are partially retained.

Fixed sample sizes at both the stages *i.e.* $n$ *psu's* and $m$ *ss'u's* within each *psu* (*say*) are considered. In case of scheme (*a*) retention pattern under consideration is the one in which $np$ *psu's* are retained on all occasions alongwith their *ssu's* and $nq$ *psu*'s are selected afresh at each occasion. In case of scheme (*b*) all *psu's* are retained alongwith only a fraction (*say*, $mp$) of *ssu's* while $mq$ *ssu's* are replaced afresh in each of the *psu's* $(p+q=1)$.

The practical utility of these schemes is evident in circumstances when repeated enquiries make the respondents more co-operative. Sometimes, the nature of information to be collected may be such that one gets better quality of data only after frequent contacts with the units. Retention of some of the sampling units over a number of occasions may then be preferred. However, in situations where response resistance developes due to repeated enquiries the schemes have limited applications. Usually for large number of occasions (say $h \geqslant 5$), response resistance may be encountered. But, it depends on the practical situation at hand and requires investigation.

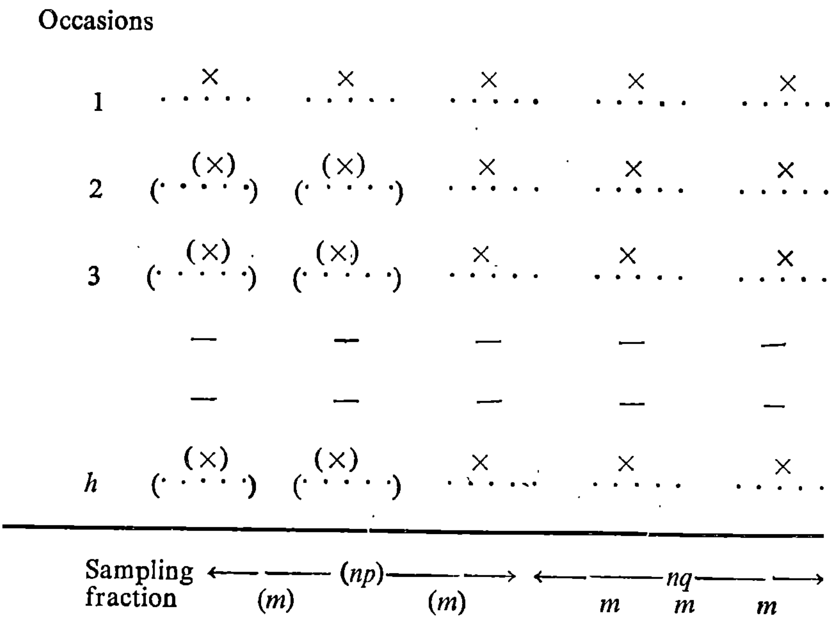
## 2. Sampling Scheme (*a*)

### 2.1 Estimate of Mean at the $t^{-th}$ Occasion When Data on $h$ Occasions are Available $(h \geqslant t)$

Consider a population consisting of $N$ *psu's* each containing $M$ *ssu's*. On the first occasion take a simple random sample (*srs*) of '$n$' *psu's* and select a *srs* of '$m$' *ssu's* from each of the selected *psu's*, selection being without replacement at each stage. On the second occasion, retain from the first occasion a sub-sample of size '$np$' of *psu's* alongwith *ssu's* and select afresh '$nq$' *psu's* from the units not selected on the first occasion. In each of the '$nq$' *psu's*, select '$m$' *ssu's* following the selection procedure as in the first occasion. The '$np$' *psu's* retained during the second occasion remain fixed for the subsequent occasions, but the remaining '$nq$' *psu's* are selected afresh in each occasion from units not selected on previous occasions.

The sample size is kept constant on each occasion keeping in view the operational convenience when actually adopted under field conditions. The sampling pattern may be expressed in the following form.

**Pattern I**




The signs '×' and '.' denote the *psu's* and *ssu's* respectively. Units inside brackets stand for the retained units, whereas those which are not in brackets denote the units taken afresh.

**Notations and definitions**

$x_{tij}$ : observation on the $j^{-th}$ *ssu* in the $i^{-th}$ *psu* at the $t^{-th}$ occasion.

$\bar{X}_t$ : population mean per *ssu* at the $t^{-th}$ occasion.

$\bar{x}_t'$ : mean per *ssu* at the $t^{-th}$ occasion for the *nmp* sampling units which are common to all the occasions.

$\bar{x}_t''$ : mean per *ssu* at the $t^{-th}$ occasion for the *nmq* sampling units which are selected afresh on the $t^{-th}$ occasion.

$\bar{X}_{ti}=\frac{1}{M}\sum_{j=1}^{M} x_{tij}$ : the mean value per *ssu* for $i^{-th}$ *psu* on the $t^{-th}$ occasion $(i=1, 2, \ldots\ldots\ldots, N)$

$$S_{bt}^2=\frac{1}{N-1}\sum_{i=1}^{N}(\bar{X}_{ti}-\bar{X}_t)^2$$

: the mean square between *psu* means in the population on the $t^{-th}$ occasion $(t=1, 2, \ldots\ldots\ldots h)$

$$S_{wi}^2=\frac{1}{N(M-1)}\sum_{i=1}^{N}\sum_{j=1}^{M}(x_{tij}-\bar{X}_{ti})^2$$

: the mean square between *ssu's* within *psu's* in the population on the $t^{-th}$ occasion $(t=1, 2, \ldots\ldots\ldots, h)$

$$\rho'_{tt'}\, S_{bt}\, S_{bt'}=\frac{1}{N-1}\sum_{i=1}^{N}(\bar{X}_{ti}-\bar{X}_t)(\bar{X}_{t'i}-\bar{X}_{t'})$$

: the true covariance between *psu* means on $t^{-th}$ and $t'^{-th}$ occasions.

$$\rho''_{tt'}\, S_{wt}\, S_{wt'}=\frac{1}{N(M-1)}\sum_{i=1}^{N}\sum_{j=1}^{M}(x_{tij}-\bar{X}_{ti})(x_{t'ij}-\bar{X}_{t'i})$$

: the true covariance between *ssu's* within *psu's* on the $t^{-th}$ and $t'^{-th}$ occasions $(t\neq t'=1, 2, \ldots\ldots\ldots, h)$

$$\alpha_t=S_{bt}^2+\frac{S_{wt}^2}{m}$$

and

$$\beta_{tt'}=\rho'_{tt'}\, S_{bt}\, S_{bt'}+\rho''_{tt'}\,\frac{S_{wt}\, S_{wt'}}{m}.$$

An unbiased linear estimates of the population mean for the $t^{-th}$ occasion utilising all the information collected upto '$h$' occasions can be written as

$$_h\bar{x}_t=\sum_{l=1}^{h} a_l\,(\bar{x}_l-\bar{x}_l'')+\bar{x}_t'') \qquad (2.1.1)$$

Considering the population to be sufficiently large and $n$ and $m$ to be relatively small such that the terms of order $n/N$ and $m/M$ can be ignored, the variance of the estimate may be written as

$$npq\ V(_h\bar{x}_t)=\sum_{l=1}^{h}\ \sum_{l'=1}^{h} a_l\, a_l'\ \gamma_{ll}'+p\alpha_t\ (1-2a_t) \qquad (2.1.2)$$

where

$$\gamma_{ll}'=q\ \beta_{ll}' \quad \text{for} \quad l\neq l'$$
$$=\alpha_l \quad \text{for} \quad l=l'$$

Optimum values of $a_l$'s ($l=1, 2 \ldots\ldots\ldots\ldots h$) are obtained, by minimising the variance $V\,(_h\bar{x}_t)$, as

$$a_l=p\ \alpha_t\frac{\Delta_h}{\Delta_h}$$

where $h$ is the determinant of matrix $((\gamma_{ll}'))$, $(1,\ 1'=1, 2\ldots\ldots\ldots h)$ ; $\gamma_{ll}'$ is the element in the $l^{-th}$ row and $l'^{-th}$ column of the matrix ; $\Delta_{tl}$ is the cofactor of $\gamma_{tl}$.

With these values of $a_l$'s, the equations (2.1.1) and 2.1.2 reduce to

$$_h\bar{x}_t=\ \sum_{l=1}^{h} p\ \alpha_t\ \frac{\Delta_{tl}}{\Delta_h}\ (\bar{x}'_l-\bar{x}''_l)+\bar{x}''_t \qquad (2.1.3)$$

and

$$V\,(_h\bar{x}_t)=\frac{\alpha_t}{nq}\left(\ 1-p\ \alpha_t\,\frac{\Delta_{tt}}{\Delta_h}\right) \qquad (2.1.4)$$

Defining

$$\delta_{ll}' = \frac{\beta_{ll}'}{\sqrt{\alpha_l \, \alpha_l'}}$$

we see that

$$\Delta_h = \left( \prod_{l=1}^{h} \alpha_l \right) \Delta'_h$$

where $\Delta'_h$ is the determinant of the matrix $((\eta_{ll}'))$ ; $(l, l'=1, 2\ldots . h)$, such that

$$\begin{aligned} \eta_{ll}' &= q\delta_{ll}' \quad \text{when} \quad l \neq l' \\ &= 1 \quad \text{when} \quad l = l' \end{aligned}$$

Now, equations (2.1.3) and (2.1.4) reduce to

$$_{h}\bar{x}_t = \sum_{l=1}^{h} p \sqrt{\frac{\alpha_t}{\alpha_l}} \frac{\Delta_{tl}'}{\Delta_h'} (\bar{x}_l' - \bar{x}_l'') + \bar{x}_t'' \qquad (2.1.5)$$

and

$$V(_{h}\bar{x}_t) = \frac{\alpha_t}{nq} \left( 1 - p \frac{\Delta_{tt}'}{\Delta_h'} \right) \qquad (2.1.6)$$

where $\Delta_{tl}'$ is the cofactor of $\eta_{tl}$. For convenience, herewith $_{h}\bar{x}_h$ is denoted by $\bar{x}_h$.

## 2.2. A Recurrence Relationship between $\bar{x}_h$ and $\bar{x}_{h-1}$

The unbiased linear estimates of the population mean at the $h^{-th}$ occasion obtained from (2.1.5) can be put as

$$\bar{x}_h = p \frac{\Delta'_{hh}}{\Delta_h'} \left\{ \bar{x}_h' + \frac{\sqrt{\alpha_h}}{\Delta_{hh}'} \sum_{l=1}^{h-1} \frac{\Delta_{hl}'}{\sqrt{\alpha_l}} (\bar{x}_l' - \bar{x}_l'') \right\} + \left( 1 - p \frac{\Delta_{hh}'}{\Delta_h'} \right) \bar{x}''_h \qquad (2.2.1)$$

Also,

$$\bar{x}_{h-1} = p \sqrt{\alpha_{h-1}} \left\{ \sum_{l=1}^{h-1} \frac{\Delta^{(h-1)'}_{h-1,\, l}}{\sqrt{\alpha_l}\, \Delta'_{h-1}} (\bar{x}_l' - \bar{x}_l'') \right\} + \bar{x}_{h-1}'' \qquad (2.2.2)$$

The superscript $(h-1)$ in $\overset{(h-1)'}{\underset{h-1,\, l}{\triangle}}$ denotes that the cofactor pertains to the determinant $\triangle'_{h-1}$. In case, no superscript is attached to a cofactor, it pertains to the determinant of $\triangle_h'$. Now, it can easily by seen that if

$$\delta_{lh}=\delta_{l,h-1}\ \delta_{h-1,\, h} \tag{2.2.3}$$

then

$$\triangle'_{hl}=p\ \delta_{h-1,\, h}\ \overset{(h-1)'}{\underset{h-1,\, l}{\triangle}},\ (l=1,\ 2\ldots\ldots,\ h-2)$$

and

$$\triangle'_{h,\, h-1}=\delta_{h-1,\, h}\ (p\ \triangle'_{h-2}-\triangle'_{h-1})$$

Therefore, from (2.2.1), it follows that

$$\bar{x}_h=p\,\frac{\triangle'_{hh}}{\triangle_h'}\left\{\bar{x}_h'+\sqrt{\frac{\alpha_h}{\alpha_{h-1}}}\ \delta_{h-1,\, h}\ (\bar{x}_{h-1}-\bar{x}_h'_{-1})\right\}+\left(1-p\,\frac{\triangle'_{h-1}}{\triangle'_h}\right)\bar{x}_h''$$

$$=a_h\left\{\bar{x}_h'+\sqrt{\frac{\alpha_h}{\alpha_{h-1}}}\ .\ \delta_{h-1,\, h}\,(\bar{x}_{h-1}-\bar{x}_h'_{-1})\right\}+(1-a_h)\ \bar{x}_h''$$

where

$$a_h=p\,\frac{\triangle_h'{}_h}{\triangle_h'}.$$

This is the same expression as obtained by Tikkiwal (1965) under the following assumptions :

$$\left.\begin{aligned}\rho'_{ij}&=\prod_{t=i}^{j=1}\rho'_{t,\, t+1}\\ \rho''_{ij}&=\prod_{t=i}^{-1}\rho''_{t,\, t+1}\end{aligned}\right\}\ \text{for all } i<j \tag{2.2.4}$$

$$\rho'_{uv}\,\frac{S_{bv}}{S_{bu}}=\rho''_{uv}\,\frac{S_{wv}}{S_{wu}}\ \text{for } u,\, v=l-1,\, l\ ;\ l=2,\ 3,\ldots h, \tag{2.2.5}$$

It can be seen that when (2.2.4) and (2.2.5) hold, the condition (2.2.3) also holds good. However, the converse is not true. Therefore, the conditions (2.2.4) and (2.2.5) are sufficient but not necessary for the above recurrence formula. It can further be seen that the set of conditions (2.2.3) are less restrictive than (2.2.4) and (2.2.5). If there are some occasions for which the assumptions (2.2.4) and (2.2.5) are not valid, the estimates can be framed as in (2.1.5) and they may be fitted in the recurrence formulae for subsequent occasions without breaking its continuity. For example, let us consider four occasions, out of which conditions during the second occasion are not normal, giving $\rho'_{12}$, $\rho'_{23}$, $\rho''_{12}$ and $\rho''_{23}$ to be very small. Obviously, the relations $\rho'_{13}=\rho'_{12}\,\rho'_{23}$ and $\rho''_{13}=\rho''_{12}\,\rho''_{23}$ are not likely to hold good in this case. As such, for third occasion, the estimate should be framed as in (2.1.5) which can be further used for making the estimate on fourth occasion by the recurrence formulae.

## 3. Sampling Scheme (*b*)

In this sampling scheme the *psu's* selected on first occasion are retained on all occasions alongwith a fraction (*say*, *mp*) of *ssu's*. In second and subsequent occasions '*mq*' *ssu's* are replaced afresh in each of the *psu's*. The sampling pattern in this case may be expressed in the following form.

**Pattern 2**

| Occasions | | | | | |
|---|---|---|---|---|---|
| 1 | × . . . . . | × . . . . . | × . . . . . | × . . . . . | × . . . . . |
| 2 | (×) (· ·)· · · | (×) (· ·)· · · | (×) (· ·)· · · | (×) (· ·)· · · | (×) (· ·)· · · |
| 3 | (×) (· ·)· · · | (×) (· ·)· · · | (×) (· ·)· · · | (×) (· ·)· · · | (×) (· ·)· · · |
| | — | — | — | — | — |
| | — | — | — | — | — |
| *h* | (×) (· ·)· · · | (×) (· ·)· · · | (×) (· ·)· · · | (×) (· ·)· · · | (×) (· ·)· · · |
| Sampling fraction | ←——— | ——— | (*n*) | ——— | ———→ |
| | (*mp*) *mq* | (*mp*) *mq* | (*mp*) *mq* | (*mp*) *mq* | (*mp*) *mq* |

All the signs have got the same meaning as in Pattern 1. Following the procedure as in section (2.1), under the same assumptions, an unbiased linear estimate of the population mean at the $t^{-th}$ occasion, utilising the information collected upto $h$ occasions is given by

$$ {}_h\bar{x}_t^* = \sum_{l=1}^{h} p\, \alpha_t^* \, \frac{D_{tl}}{D_h} \, (\bar{x}_l' - \bar{x}_l'') + \bar{x}_t'' \tag{3.1} $$

and its variance is given by

$$ V({}_h\bar{x}_t^*) = \frac{\alpha_t^*}{nq} \left( 1 - p\, \alpha_t^* \, \frac{D_{tt}}{D_h} \right) + \frac{S_b^2{}_t}{n} \tag{3.2} $$

where $D_h$ is the determinant of the matrix $((\gamma^*_{ll'}))$, $(l,\ l'=1,\ 2 \ldots,)$ such that

$$ \gamma^*_{ll'} = q\, \beta^*_{ll'} = q\, \rho_{ll'}'', \ \frac{S_{wl}\, S_{wl'}}{m} \qquad \text{when} \qquad l \neq l' $$

$$ = \alpha_l^* = \frac{S_{wl}{}^2}{m} \qquad \text{when} \qquad l = l' $$

and $D_{tl}$ is the cofactor of $\gamma^*_{tl}$ in $D_h$. The estimate and its variance can be put in a convenient form as

$$ {}_h\bar{x}_t^* = \sum_{l=1}^{h} p\, \frac{S_{wt}}{S_{wl}} \, \frac{D_{tl}'}{D_h'} \, (\bar{x}_l' - \bar{x}_l'') + \bar{x}_t'' \tag{3.3} $$

and

$$ V({}_h\bar{x}_t^*) = \frac{S^2{}_{wt}}{nmq} \left( 1 - p\, \frac{D'_{tt}}{D_h'} \right) + \frac{S^2{}_{bt}}{n} \tag{3.4} $$

where $D_{tl}$ is the cofactor of $\eta^*_{tl}$ in $D_h'$ ; $D_h'$ being the determinant of matrix $((\eta^*_{ll'}))$ $(l,\ l'=1,\ 2 \ldots\ldots h)$

$$ \eta^*_{ll'} = q\, \rho''_{ll'} \qquad \text{when} \qquad l \neq l' $$

$$ = 1 \qquad \text{when} \qquad l = l $$

If

$$\rho''_{lh} = \rho''_{l,\, h-1}\ \rho''_{h-1,\, h}\,,\ (l=1,\, 2, \ldots\ldots h-2) \tag{3.5}$$

the estimate of mean as obtained from (3.3) can be put as

$$\bar{x}_h^* = p\,\frac{D'_{hh}}{D_h'}\left\{\bar{x}_h' + \frac{S_{wh}}{S_{wh-1}}\ \rho''_{h-1,\,h}\,(\bar{x}_{h-1} - x'_{h-1})\right\} + \left(1 - p\,\frac{D_{hh}'}{D_h'}\right)\bar{x}_h''$$

## 4. Efficiencies Of The Estimates

Comparison of the estimates for general pattern of correlation was, however, not possible. As such, comparisons have been made under the exponential correlation model $\delta_{ll'} = \delta^{|l-l'|}$ for all l and $l'$ $(l \neq l')$. Table 1 gives the percentage gain in efficiency of the mean $\bar{x}_h$ over $\bar{x}_h^*$ for different values of $h, q, \rho', \rho''$ and $\phi/m \left(= \frac{S_w^2}{mS_b^2}\right)$. Some conclusions are as follows :

(*i*) The percentage gain in efficiency '$G$' increase as $\frac{\varphi}{m}$ decreases for all sets of values of $h$, $q$, $\rho'$ and $\rho''$.

(*ii*) '$G$' increases as $\rho'$ increases for fixed $h$, $q$, $\frac{\varphi}{m}$ and $\rho''$.

(*iii*) For a given set of $h, q, \frac{\varphi}{m}$ and for

$\rho' = .5$, '$G$' decreases consistently for increasing values of $\rho''$.
$\rho' = .7$, '$G$' is maximum at $\rho'' = .7$ and
$\rho' = .9$, '$G$' increases consistently for increase in $\rho''$.

(*iv*) Sampling scheme (*a*) is more efficient than sampling scheme (*b*) for the estimate of mean for almost all values of $h$, $q$, $\frac{\varphi}{m}$, $\rho'$ and $\rho''$ with a few exceptions. These sets are given by the negative values of '$G$' in the table.

A similar comparison of efficiency is made by Kathuria and Singh (1971). Percentage gain in efficiency of $\bar{x}_h$ over $\bar{x}_h^*$ for the model $\delta_{ll'} = \delta$ have also been worked out and it provides similar conclusions. However, the numerical values of gains in efficiency are increased consistently. An overall indication from this study is that the sampling scheme (*a*) should be preferred to the scheme (*b*).

5. Robustness Of The Estimates

In this section robustness of the estimates are studied with respect to slight to moderate changes from exponential model of correlations. Two models considered for this purpose are :

**Model 1 :** Exponential correlation model as $\delta_{ij}=\delta^{|i-j|}$.

**Model 2 :** This is an arithmetic correlation model similar to that of Hansen *et. al.* [1955] and Rao and Graham [1964].

$$\delta_{ij}=\delta+(1-|i-j|)d \quad \text{for} \quad \delta+(1-|i-j|)\,d>0$$
$$=0 \quad \text{otherwise}$$

Table-2 has been prepared using an *IBM* 1620. For sampling scheme (*a*), table-2 provides the percentage gain in efficiency of estimates $\bar{\bar{x}}_h$ ($h=3$, 4 and 5) over simple random sampling and the optimum $q$ for the values of $d=0.05\ (0.05)\ 0.20$, $\delta=0.5(0.1)\ 0.9$ for model-2 and same values of $\delta$ for model 1. It is seen from this table that model-1 is fairly robust for slight to moderate deviations as shown by the alternate arithmetic model. However, with increasing values of $\delta$ and $h$ the robustness shows a decreasing trend. Optimum $q$'s for models 1 and 2 are either same or deviate only by 0.1.

Investigations for sampling scheme (*b*) also provide similar conclusions regarding robustness.

It may further be mentioned that calculation of optimum replacement fraction for correlation models 1 and 2 is not straightforward. It has been obtained with the help of electronic computer. However, when $\delta_{it}=\delta$ the calculation of optimum replacement fraction becomes quite simple and is given by the positive root of the quadratic equation.

$$\delta\,\{(h-1)\,\delta-(h-2)\}\,q^2-2q+1=0$$

Thus, the optimum replacement fraction is

$$_hq_0=\frac{1}{1+\sqrt{(1-\delta)^2+h\delta\,(1-\delta)}}$$

## SUMMARY

In the present paper the theory of successive sampling for two stage sampling designs has been discussed for following two sampling schemes :

(*a*) Primary Sampling Units (*p.su's*) are partially retained alongwith their secondary sampling units (*ssu's*).

(*b*) All *psu's* are retained and *ssu's* are partially retained.

It is observed that in most of the practical situations sampling scheme (*a*) should be preferred over sampling scheme (*b*).



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## REFERENCES


1. Abraham, T.P., Khosla, R.K. and Kathuria. O.P. (1969) : "Some investigations on the use of successive sampling in pests and diseases surveys", Jour. Ind. Soc. Agri. Stat., Vol. 21, No. 2.
2. Eckler. A.R. (1955) : "Rotation Sampling", Ann. Math. Stat. Vol. 26.
3. Jessen R.J. (1942) : "Statistical Investigations of a sample survey for obtaining farm facts", Iowa Agri. Exp. Stn. Res. Bull. 304.
4. Kathuria, O.P. (1959) : "Some aspects of successive sampling in multistage designs", Thesis submitted for award of I.C.A.R. Diploma (Unpublished).
5. Kathuria, O.P. and Singh, D. (1971) : "Comparisons of estimates in two stage sampling on successive occasions", Jour. of Ind. Soc. of Agri. Stat., Vol. 23, No. 1.
6. Patterson, H.D. (1950) : "Sampling on successive occasions with partial replacement of units", Jour. Roy. Stat. Soc., Ser. B., 12.
7. Prabhu-Ajgaonkar, S.G." (1968) : The theory of univariate sampling on successive ocsasions under the general correlation pattern", Austr. Jour. of Stat.. Vol. 10, No. 2.
8. Rao, J.N.K. and Graham, J.E. (1964) : "Rotation designs for sampling on repeated occasions", Jour. of Roy. Stat. Soc., Vol. 59.
9. Singh, D. (1968) : "Estimates in successive sampling using a multistage design", Jour. of Amer. Stat. Assoc., 63.
10. Singh, D. and Kathuria, O.P. (1969) : On two stage successive sampling", Austr. Jour. of Stat. Vol. 21 No. 2.

11. Singh, S. (1970) : "Some studies in two stage successive sampling", thesis submitted for award of I.C.A.R. Diploma (unpublished).

12. Tikkiwal, B.D. (1955) : "Multiphase sampling on successive occasions", thesis submitted to the faculty of North Carolina State College, U.S.A., in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

13. Tikkiwal, B.D. (1951) : "Theory of successive sampling", thesis submitted for award of I.C.A.R. Diploma (unpublished).

14. Tikkiwal, B.D. (1956) : "A further contribution to the theory of univariate sampling on successive occasions", Jour of Ind. Soc. Agri. Stat., 8.

15. Tikkiwal, B.D. (1960) : "On the theory of classical regression and double sampling estimation", Jour. Roy. Stat. Soc.. Ser. B., Vol. 22.

16. Tikkiwal, B.D. (1964) : "A note on two stage sampling on successive occasions", Sankhya, Series A., Vol. 26, Part I.

17. Tikkiwal, B.D. (1965) : "The theory of two-stage sampling on successive occasions", Jour. Ind. Stat. Assoc.

18. Yates, F. (1949) : "Sampling Methods for Census and Surveys", Charles Griffin and Company Ltd., London.

TABLE 1

**Percentage gain in efficiency of $\bar{x}_h$ over $\bar{x}_h{}^*$ under exponential model, $\delta_{ij}=\delta^{|i-j|}$**

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $h$ | $\rho'$ | $\Phi/m$ | $q=0.25$ | | | $q=0.50$ | | | $q=0.75$ | | |
| | | | $\rho''\to 0.5$ | 0.7 | 0.9 | 0.5 | 0.7 | 0.9 | 0.5 | 0.7 | 0.9 |
| | 0.5 | 0.1 | 4.50 | 4.48 | 4.27 | 6.49 | 6.38 | 5.89 | 5.24 | 5.07 | 4.16 |
| | | 1.0 | 2.49 | 2.32 | 1.63 | 3.57 | 3.22 | 1.46 | 2.88 | 2.31 | −1.45 |
| | | 2.0 | 1.66 | 1.51 | 0.87 | 2.38 | 2.06 | 0.35 | 1.92 | 1.39 | −2.55 |
| 2 | 0.7 | 0.1 | 9.38 | 9.51 | 9.48 | 14.44 | 14.75 | 14.66 | 12.82 | 13.20 | 12.83 |
| | | 1.0 | 4.86 | 5.23 | 5.14 | 6.27 | 8.11 | 7.84 | 6.26 | 7.26 | 5.90 |
| | | 2.0 | 3.16 | 3.49 | 3.41 | 4·69 | 5.40 | 5.16 | 4.00 | 4.84 | 3.52 |
| | 0.9 | 0.1 | 16.68 | 17.08 | 17.31 | 28.95 | 30.13 | 30.93 | 31.08 | 33.42 | 35.17 |
| | | 1.0 | 7.83 | 8.86 | 9.52 | 12.35 | 14.90 | 17.01 | 11.40 | 15.27 | 19.34 |
| | | 2.0 | 4.91 | 5.78 | 6.34 | 7.54 | 9.56 | 11.34 | 6 72 | 9.58 | 12.89 |
| | 0 5 | 0.1 | 5.24 | 5.11 | 4.49 | 6.99 | 6.80 | 5.79 | 5.36 | 5.16 | 3.85 |
| | | 1.0 | 2.87 | 2.44 | 0.15 | 3.84 | 3.21 | −0.94 | 2.94 | 2.23 | −3.57 |
| | | 2.0 | 1.91 | 1.51 | −0.73 | 2.57 | 1.98 | −2.37 | 1.96 | 1.30 | −5.09 |
| 3 | 0.7 | 0.1 | 12.41 | 12.73 | 12.56 | 17.07 | 17,60 | 17.27 | 13.63 | 14.11 | 13.46 |
| | | 1.0 | 6.16 | 7.00 | 6.43 | 8.31 | 9.68 | 8.51 | 6.55 | 7.76 | 5.19 |
| | | 2.0 | 3.95 | 4.67 | 4.14 | 5.31 | 6.45 | 5.32 | 4.16 | 5.18 | 2.55 |

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.9 | 0.1 | 26.90 | 28.27 | 29.18 | 41.40 | 44.48 | 46.90 | 37.49 | 41.47 | 44.92 |
| | | 1.0 | 10.89 | 13.71 | 16.05 | 15.10 | 20.16 | 25.80 | 12.30 | 17.55 | 24.70 |
| | | 2.0 | 6.54 | 8.75 | 10.70 | 8.90 | 12.62 | 17.20 | 7.13 | 10.83 | 16.47 |
| | 0.5 | 0.1 | 5.35 | 5.16 | 4.23 | 7.03 | 6.81 | 5.53 | 5.37 | 5.15 | 3.75 |
| | | 1.0 | 2.94 | 2.32 | −1.45 | 3.86 | 3.14 | −2.43 | 2.95 | 2.23 | −4.13 |
| | | 2.0 | 1.96 | 1.39 | −2.52 | 2.58 | 1.89 | −4.14 | 1.97 | 1.28 | −5.81 |
| 4 | 0.7 | 0.1 | 13.32 | 13.73 | 13.38 | 17.50 | 18.09 | 17.58 | 13.68 | 14.16 | 13.42 |
| | | 1.0 | 6.47 | 7.54 | 6.31 | 8.44 | 9.94 | 7.96 | 6.57 | 7.79 | 4.77 |
| | | 2.0 | 4.13 | 5.03 | 3.84 | 5.37 | 6.63 | 4.63 | 4.17 | 5.19 | 2.07 |
| | 0.9 | 0.1 | 32.59 | 34.98 | 36.73 | 45.91 | 50.25 | 53.97 | 38.55 | 43.02 | 47.08 |
| | | 1.0 | 11.91 | 16.03 | 20 20 | 15.59 | 21.66 | 29.67 | 12.36 | 17.82 | 25.89 |
| | | 2.0 | 7.00 | 10.06 | 13.47 | 9.09 | 13.41 | 19.78 | 7.15 | 10.95 | 17.26 |
| | 0.5 | 0.1 | 5.36 | 5.16 | 4.02 | 7.03 | 6 82 | 5.42 | 5.37 | 5.15 | 3.72 |
| | | 1.0 | 2.94 | 2.26 | −2.59 | 3.86 | 3.12 | −3.10 | 2.95 | 2.23 | −4.26 |
| | | 2.0 | 1.96 | 1.32 | −3.86 | 2.58 | 1.87 | −4.99 | 1.97 | 1.28 | −5.99 |
| 5 | 0.7 | 0.1 | 13.58 | 14.03 | 13.53 | 17.56 | 18.18 | 17.56 | 13.68 | 14.16 | 13.40 |
| | | 1.0 | 8.54 | 7.72 | 5.86 | 8.45 | 10.00 | 7.54 | 6.57 | 7.79 | 4.69 |
| | | 2.0 | 4.16 | 5.15 | 3.31 | 5.38 | 6.66 | 4.14 | 4.17 | 5.19 | 1.94 |
| | 0.9 | 0.1 | 35.61 | 38.79 | 41.31 | 47.45 | 52.42 | 56.90 | 38.72 | 43.31 | 47.54 |
| | | 1.0 | 12.23 | 17.07 | 22.72 | 15.68 | 22.08 | 31.29 | 12.36 | 17.89 | 26.14 |
| | | 2.0 | 7.11 | 10.61 | 15.14 | 9.12 | 13.60 | 20.86 | 7.15 | 10.96 | 17.43 |

TABLE 2

**Percent gain in efficiency of $\bar{x}_h$ over sample mean and optimum $q$ (in parenthesis) under the exponential model $\delta_{ij}=\delta^{|i-j|}$**

| $h$ | $\delta \rightarrow$ / $d \downarrow$ | .5 | .6 | .7 | .8 | .9 |
|---|---|---|---|---|---|---|
| 3 | .05 | 11.07(.5) | 17.13(.5) | 25.58(.5) | 38.19(.6) | 59.56(.7) |
| | .10 | 9.90(.5) | 15.47(.5) | 23.19(.5) | 34.33(.6) | 52.92(.6) |
| | .15 | 8.96(.5) | 14.12(.5) | 21.23(.5) | 31.32(.6) | 47.92(.6) |
| | .20 | 8.23(.5) | 13.02(.5) | 19.64(.5) | 29.01(.6) | 44.17(.6) |
| | Model 1 | 7.69(.5) | 12.33(.5) | 19.37(.5) | 30.81(.6) | 54.10(.6) |
| 4 | .05 | 13.13(.5) | 20.43(.5) | 30.68(.5) | 45.64(.5) | 70.37(.6) |
| | .10 | 10.70(.5) | 16.92(.5) | 25.58(5) | 37.97(.5) | 57.15(.6) |
| | .15 | 9.11(.5) | 14.15(.5) | 22.08(.5) | 32.77(.5) | 49.16(.6) |
| | .20 | 8.24(.5) | 13.04(.5) | 19.78(.5) | 29.29(.5) | 44.29(.6) |
| | Model 1 | 7.73(.5) | 12.48(.5) | 19.90(.5) | 32.68(.5) | 59.94(.6) |
| 5 | .05 | 14.19(.4) | 22.08(.5) | 33.28(.5) | 49.73(.5) | 75.75(.5) |
| | .10 | 10.79(.5) | 17.20(.5) | 26.14(.5) | 38.95(.5) | 58.30(.5) |
| | .15 | 9.21(.5) | 14.55(.5) | 22.09(.5) | 32.81(.5) | 49.16(.6) |
| | .20 | 8.32(.5) | 13.55(.5) | 20.11(.5) | 29.56(.6) | 44.85(.5) |
| | Model 1 | 7.73(.5) | 12.50(.5) | 20.00(.5) | 33.17(.5) | 62.58(.5) |